Maximization principles for lower semicontinuous functions and the Banach-Mazur game

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Abstract

A typical variational principle in optimization studies minimization of perturbations of a bounded from below lower semicontinuous function by elements from a prescribed family of functions, e.g., by linear continuous functions (Bishop-Phelps theorem, Stegall principle), by scalar modifications of the norm (Ekeland variational principle, Borwein-Preiss variational principle), by continuous functions (Choban, Kenderov, Revalski) etc.. Symmetrically, one can study the same problem for maximization of a bounded from above upper semicontinuous function. In this talk we will study variational principles for **maximization** of perturbations of **a lower semicontinuous** function (not for minimization of such functions which is the usual case).

It turns out that the existence of a winning strategy for one of the players in the Banach-Mazur game played in the underlying space characterizes the validity of a generic variational principle for existence of solutions for the maximization of continuous bounded perturbations of a fixed lower semicontunuous function which is bounded from above. We give also some consequences of this result which give another (internal) characterizations for the underlying space in the case of the validity of the above principle. Attention is paid also to the case when the perturbations not only possess a maximum, but the corresponding optimization problem is well-posed. We also provide examples which outline the limits for having the validity of such principles.

The talk is based on joint work with M. Ivanov and P. Kenderov from the paper [1]

References

[1] IVANOV, M., KENDEROV, P.S. & REVALSKI, J.P., Variational principles for maximization problems with lower-semicontinuous goal functions, *Set-valued and Variational Analysis* **30**: 559–571, 2022.

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