The Steiner Selection: Geometry, Stability, and Sensitivity

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Abstract

The Steiner point is a classical geometric construction that associates a canonical point with a convex body. Introduced by Steiner in the nineteenth century, it plays an important role in convex geometry and has found applications in set-valued analysis, variational analysis, and the theory of differential inclusions.

In this talk we investigate the geometry, stability, and sensitivity of the Steiner selection for families of convex bodies depending on parameters. We first establish stability-transfer principles showing that regularity properties of convex-valued mappings propagate to their Steiner selections, yielding Lipschitz stability under Hausdorff perturbations.

We then analyze the differential structure of the Steiner selection. Under mild assumptions we establish almost-everywhere differentiability and derive an explicit Jacobian formula describing the first-order variation of the Steiner selection in terms of directional derivatives of the support function. This representation reveals directional sparsity and rank properties of the Jacobian, leads to anisotropic Lipschitz estimates, and, under stronger assumptions, yields a $C^{1,1}$-type regularity result.

We also discuss the polyhedral setting, where the general formulas reduce to finite expressions involving vertex data and show how localized changes in the normal fan influence the variation of the Steiner point. Finally, we consider smooth deformations of convex bodies and derive first- and second-order variation formulas, and briefly comment on connections with other regularization procedures used in nonsmooth analysis.